Compressed Sensing for Multistatic Radar Systems with Arbitrary Block Codes

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***Abstract*— Multistatic radar systems with multiple elements at transmitting and/or receiving side permit multiple target observations from different viewpoints providing various advantageous. In an environment with multiple transmitters the emitters may transmit independent waveform or use a coding structure to emit pulses across antennas over a predetermined coherent time-interval. In this work we extend the principles of employing orthogonal block codes for this purpose and emphasize on arbitrary or randomly designed block codes with a compressed sensing based detection at receiver. By employing a compressed sensing approach only a subset of incoming data needs to be stored and processed thus significantly reducing the sampling and integration requirement at the receivers. We show that the use of more general type of block codes allows for greater flexibility and better performance in target detection and are hence more suitable for compressed sensing methods than orthogonal, or quasi-orthogonal, block codes. Through simulations it is validated that such a system can operate well and compressed sensing techniques allow for data reduction and improved detection with a shorter dwell period.**

I. Introduction

Multistatic or MIMO (multiple-input multiple-output) systems with widely distributed antennas exploit angular and spatial diversity to provide numerous advantages. Each antenna typically emits a different waveform thus the receiving antenna has the potential to observe a target from a varying numbers of aspect angles. This can be used to for example recede target fading and provide enhancements in target detection, directional finding etc. [1], [2], [3]. For these techniques to operate an often made assumption is that the waveforms emit from antennas are all orthogonal to each others. In reality, this condition is hard to satisfy and as an alternative space-time block coding methods may be put to use with non-orthogonal waveforms [4], [5], [6]. The waveforms are then emit across antennas and time slots in a specific manner where the given coding and receiver processing ensures signal separation with full diversity by simple matched filtering. The main benefit of such an approach is that the waveforms can be designed freely. More specifically one may utilize randomly or chaotic generated noise waveforms, which are of importance for military radar systems.

This paper proposes a radar system model to take advantage of block codes for multistatic transmission and as incoming radar signals are typically sparse in nature, employ a data reduction strategy at the receiver and processing end. Instead of the receiving antennas continuously collecting data with respect to incoming signals empty gaps of varying lengths can be incorporated either at random or in a predetermined manner. The coding duration of the block code emit may also be reduced at the transmitter. To recover the signals compressed sensing (CS) algorithms [7], [8], [9], [10], [11] can be applied to locate and detect potential targets. We show that such an approach has considerable merit as the sampling requirements at the receiver antennas are reduced and the block integration time can be altered. For certain signals, such as noise waveforms, the signal recovery can be shown to be exact. The use of orthogonal block codes was introduced in [12], however, as we show here orthogonal and quasi-orthogonal block codes designed primarily for matched filter based decoding are non-optimal when combined with a compressed sensing based detection. Designing arbitrary block codes also allow for much greater flexibility is determining the coherent block time interval and results in lower mutual coherence. Otherwise selecting orthogonal, or to some extent quasi-orthogonal, block-codes could be expected to be a good starting point for compressed sensing applications as subsampled orthogonal structures can yield effective sensing matrices for signal recovery [13].

II. Generic signal model

A multistatic radar setup with M transmitting and a single receiving antenna is assumed, though the system is easily extendable to the case of multiple receiving antennas. The transmitter uses a space-time block code $\mathbf{C}$ as a model to transmit waveforms where the columns of $\mathbf{C}$ denotes the antennas and the rows the time slots. The block code is assumed to require an integration period of K slots. The incoming signal at the receiver over the total time duration of $K \times T$, can then be expressed as

$$\mathbf{r}(t) = \begin{bmatrix} r_1(t) \\ : \\ r_K(t) \end{bmatrix} = \mathbf{C} * \begin{bmatrix} \rho_1(t) \\ : \\ \rho_M(t) \end{bmatrix} = \mathbf{C} * \rho(t) + n(t), \quad (1)$$

where $r_i(t)$ corresponds to incoming data at the respective transmissional slot (row i) of $\mathbf{C}$ which is collected over the time duration T. Similarly, block $\rho_i(t)$ refers to the echo reflectively, incorporating target's radar cross-section, propagation etc. originating through transmitter antenna i. Each element in $r_i(t)$ consequently corresponds to a range-bin $\Delta, 2\Delta, 3\Delta, ..., N\Delta$ where Δ is the range-bin resolution and

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$T\Delta$ the maximum antenna range. $*$ denotes the convolutional operator and $n(t)$ is additive noise. We assume no or marginal Doppler within the dwell period of the code.

In a traditional processing approach a matched filtering operation is carried over $\mathbf{r}(t)$ with the conjugated version of the code, $\mathbf{C}^*$, to recover potential targets as orthogonal block codes fulfill the property $\mathbf{C}^*\mathbf{C} = \alpha\mathbf{I}$ where $\mathbf{I}$ is the identity matrix and $\alpha = \sum s_i(t)s_i^*(t)$ corresponds to the waveforms reflecting the overall system's pulse spreading function. However, as orthogonal or quasi-orthogonal, codes are generally designed and optimized for linear matched filter based processing it is unlikely that they are still highly suitable when other types of receiver processing is to be performed.

Signal model (1) may be re-written in the form of a matrix multiplication through the aid of a convolutional matrix $\mathbf{S}$,

$$\mathbf{r}(t) = \mathbf{S}\ \rho(t) + n(t). \tag{2}$$

The matrix $\mathbf{S}$ follows a structure with Toeplitz blocks consisting of delay shifted waveforms. The incoming radar data is however likely to be highly sparse and contain many zero figures pointing towards the fact that all matrix entries may not be decisive for a signal recovery particularly when there may only be a few targets to detect. To contract the amount of data, the receiver may thus decide to cut down on sample storing in the spirit of compressed sensing methodology. This can be accomplished by e.g. not sampling with respect to certain range-bins during alternating block code transmission periods. As an example amid the reception of the first row of the block code the receiver may decide to store incoming data corresponding to range-bins $2\Delta, 4\Delta, 6\Delta, ...$ while during the second row transmission sample and store with respect to range-bins $\Delta, 3\Delta, 5\Delta, , ...$. This trims the amount of data in half. This process can also be performed randomly and detailed conventionally through a sensing matrix $\mathbf{\Phi}$. The observed data is then described by

$$\mathbf{r}(t) = \mathbf{\Phi}\ \mathbf{S}\ \rho(t) + n(t). \tag{3}$$

A plain sensing matrix $\mathbf{\Phi}$ will therefore contain either 0 or 1 corresponding to the observation at that time step and range-bin is stored or discarded.

Beside not sampling with respect to certain ranges, the receiver may further decide not to store data altogether with respect to given transmissional periods. This is analogous to not transmitting a row of the block code and cuts down on required block transmissional period; which may be quite lengthy if a large number of transmitters are put to use. This does eliminate the orthogonal properties of an orthogonal block code but since a CS detection does not rely directly on that, one may study various approaches like this to curtail data intake.

By eliminating any empty row entries of $\mathbf{\Phi S}$ and the corresponding values of $\mathbf{r}(t)$ we can re-write (3) as

$$\hat{\mathbf{r}}(t) = \hat{\mathbf{S}}\ \rho(t) + n(t). \tag{4}$$

The under-determined linear system of (4) generally does not have a single unique solution, however, a recovery process through CS can lead to the desired unique solution if the matrix $\hat{\mathbf{S}}$ satisfies certain conditions [7], [8]. One such condition is that the mutual coherence μ of $\hat{\mathbf{S}}$ is low, defined as

$$\mu = \max_{i \neq j} \frac{|\hat{S}_i^* \hat{S}_j|}{||\hat{S}_i|| ||\hat{S}_j||}. \tag{5}$$

The number of targets for whom perfect reconstruction is guaranteed is then given by k, $k < \frac{1}{2}(\mu^{-1} + 1)$. Assuming the elements of $\mathbf{\Phi}$ are selected randomly off-line, or in a pseudo-randomly manner, the matrix $\hat{\mathbf{S}}$ can be designed and optimized beforehand with respect to the given waveforms and the desired data rate to achieve a low mutual coherence value. We point out that if elements of $\mathbf{\Phi S}$ are selected randomly, i.e. from a noise waveform, and optionally follow a Toeplitz or block structure [14], [15], [16], then the matrix will typically result in acceptable CS conditions for signal recovery [17]. In this particular case the matrices are sparse convolutional Toeplitz blocks which can through permutation methods e.g. Cuthill-McKee type algorithms [18] generally be converted into a matrix with a block diagonal structure. An example of this is shown in figure 1 for a matrix used in the next section where the bipartite graphs of the original matrix and the permuted matrix are displayed. Although the permuted matrix retains some sparsity it nevertheless shows a very concise diagonal structure. With very low data levels the matrix properties will still start to deviate too strongly to satisfy a perfect recovery; how many targets may be detected as a function of data rate is examined in the simulation section.

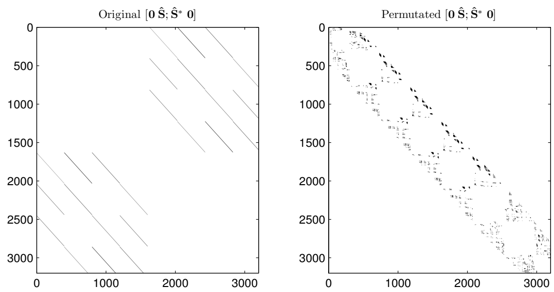

Fig. 1. A convolutional matrix and it's permuted version

The recovery of $\rho(t)$ can be accomplished by well-established methods by e.g. attempting to find $\min\ ||\tilde{\rho}(t)||_1$ subject to $\hat{\mathbf{S}}\ \tilde{\rho}(t) = \hat{\mathbf{r}}(t)$, which is particularly suitable for low-noise situations. In the presence of noise the minimization can be carried out with subject to $||\hat{\mathbf{S}}\ \tilde{\rho}(t) - \hat{\mathbf{r}}(t)||_2 \leq \epsilon$. Numerous algorithms have been proposed in the literature [9].

III. Four transmitter case

We next portray an example case of the proposed approach and demonstrate it through simulations. A four transmitter antenna case is considered with a single receiver antenna. For this scenario we put to use a more or less randomly selected block code with no specific properties apart from the fact that

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each transmitter emits all four pulses across the four time slots:

$$\mathbf{C}_4 = \frac{1}{\sqrt{4}} \begin{pmatrix} s_1(t) & s_2(t) & s_3(t) & s_4(t) \\ s_2^*(t) & -s_3(t) & s_4^*(t) & -s_1(t) \\ -s_4(t) & s_1(t) & s_2^*(t) & -s_3(t) \\ s_3^*(t) & -s_4^*(t) & -s_1^*(t) & s_2(t) \end{pmatrix}. \quad (6)$$

The second code we analyze is based on emitting different pulses on all occasions, corresponding to a radar system where no coding is done across time:

$$\mathbf{C}_N = \frac{1}{\sqrt{4}} \begin{pmatrix} s_1(t) & s_2(t) & s_3(t) & s_4(t) \\ s_5(t) & s_6(t) & s_7(t) & s_8(t) \\ s_9(t) & s_{10}(t) & s_{11}(t) & s_{12}(t) \\ s_{13}(t) & s_{14}(t) & s_{15}(t) & s_{16}(t) \end{pmatrix}. \quad (7)$$

To compare these against more established block codes we use the orthogonal code [19]

$$\mathbf{C}_O = \frac{1}{\sqrt{3}} \begin{pmatrix} s_1(t) & s_2(t) & s_3(t) & 0 \\ -s_2^*(t) & s_1^*(t) & 0 & s_3^*(t) \\ -s_3^*(t) & 0 & s_1^*(t) & -s_2(t) \\ 0 & -s_3(t) & s_2^*(t) & s_1(t) \end{pmatrix}, \quad (8)$$

and the quasi-orthogonal code of [20]:

$$\mathbf{C}_q = \frac{1}{\sqrt{4}} \begin{pmatrix} s_1(t) & s_2(t) & s_3(t) & s_4(t) \\ -s_2^*(t) & s_1^*(t) & -s_4^*(t) & s_3^*(t) \\ -s_3^*(t) & s_4(t) & s_1^*(t) & -s_2(t) \\ -s_4^*(t) & -s_3(t) & s_2^*(t) & s_1(t) \end{pmatrix}. \quad (9)$$

$s_1(t)$ to $s_{16}(t)$ refers to the different waveforms of the block code with $s_i^*(t)$ being the conjugated waveform. Although the orthogonal block code only emits 3 independent waveforms it nevertheless satisfies the major property of $\mathbf{C}_O^*\mathbf{C}_O = \alpha\mathbf{I}$. The receiver will hence be able to determine which targets are arriving through which emitter without any ambiguity or interference with a matched filtering operation over $\mathbf{r}(t)$ as long as full data set is acquired - which may be excessive and unnecessary assuming that there may only be a few targets to detect.

The single receiving antenna is simulated to receive a total of 12 targets of varying amplitudes, with 4 target echos off transmitter antenna one, 2 from the second antenna and 3 returns from antenna three and four, as depicted in figure 2. The simulated range is composed of 400 sampling bins. How

Fig. 2. Simulated scenario of target returns

well a CS recovery process performs in this given scenario will depend primarily on parameters such as the pulse length, number of targets and collected data amount.

For the simulations we consider cases with varying amount of pulse lengths and investigate the performance of the recovery process. This is accomplished with the block codes as given in (6)-(9) but where only the first two, or the first three, transmissional periods are put to use. This cuts the block transmissional period by, respectively, half and one-quarter. In addition to that the receiver utilizes a randomly generated, unoptimized, sensing matrix to only collect and store data at certain sampling ranges in a typical random compressed sensing framework. Noisy baseband waveforms are generated and for the first simulations consist of only 6 discrete samples. Standard compressed sensing algorithms from are utilized to obtain target location estimates.

A typical data reconstruction example is shown in figure 3 with orthogonal block code $\mathbf{C}_O$ and transmission/reception of only first three rows of the code with a further 50% random data reduction via the sensing matrix assuming and average SNR of 8dB. Although not a perfect reconstruction due to the heavy data cutback, apart from the weakest target returns, majority of reflections come out quite clear and are, due to absence of noise, easily detectable through a simple thresholding test.

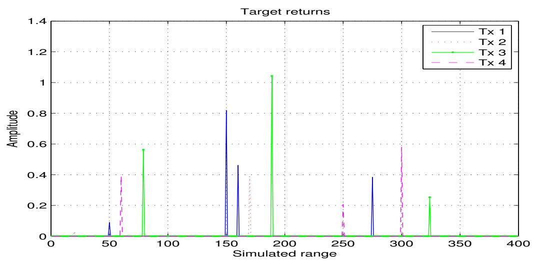

Fig. 3. CS based reconstruction with 50% deduction

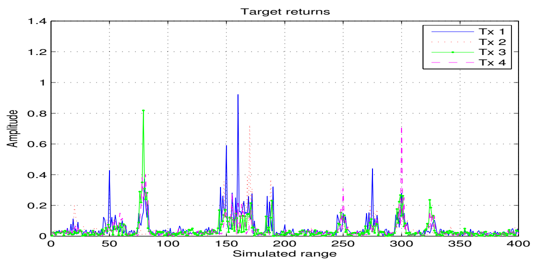

Fig. 4. Matched filter based reconstruction with 50% deduction

A standard matched filtering output on the same signal data provides figure 4, were the overall SNR is lower and where the impacts of the pulse spreading functions are much more discernible. A CFAR type detector would need to be applied for target detection, more importantly, matched filtering in this situation does not have the ability to distinguish the waveforms which causes significant overlaps of spreading functions particularly at the peak points; thus making it difficult to to locate and determine with accuracy which reflections arrive through which transmitter.

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More sophisticated detection algorithms can combine the results from matched filtering, which is computationally inexpensive, with CS to reduce the overall false detection rate. These simulations do demonstrate the fact that CS methods under space-time block codes can work exceptionally well in certain settings and the increased system complexity can to some extent be compensated by reducing the block dwell period and relying on collection of randomized data. Similar methods can also become useful when only parts of the data may be available in e.g. a distributed environment.

More detailed simulations are shown in figures 5 and 6 where the sampling sensing matrix is randomly varied to alter the collected data rate. The plots display the results of the average number of targets detected through a thresholding test, out of a total of 12 targets, based on 100 simulation runs. This is shown as a function of the reduced data rate, in percentage, under the prescribed block code consisting of either 3 or 2 block integration intervals with the average SNR at 8dB and each pulse composed of six samples.

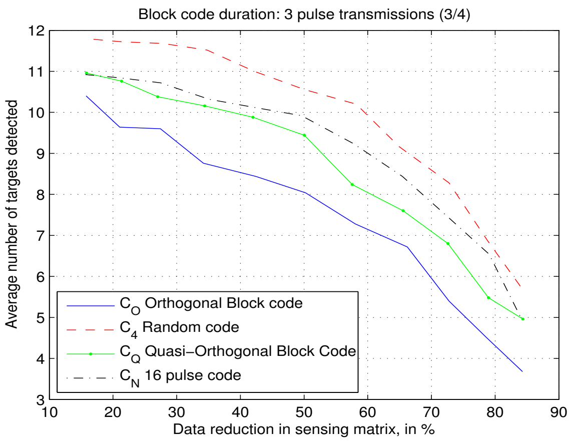

Fig. 5. Target detection with CS, 3 block code rows, SNR=8dB, $p_{len} = 6$

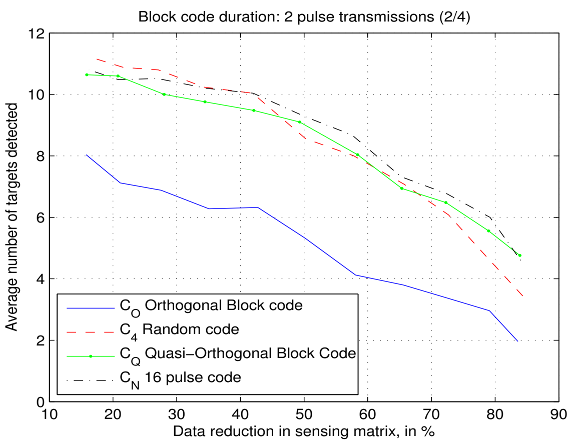

Fig. 6. Target detection with CS, 2 block code rows, SNR=8dB, $p_{len} = 6$

It is evident from the simulations that the arbitrary block code (6) gives reasonable outcomes while the orthogonal block code clearly starts to lag behind. Performance of $\mathbf{C}_N$ with all independent pulses is also respectable along with quasi-orthogonal code under limited integration time. In all cases, the weaker targets are more easily degraded, particularly in low SNR situations, while the more stronger ones remain stable for a recovery in a CS reconstruction. Heavy data degradation starts to cause more issues with fewer targets being detected, particularly where only two rows of the block code are put to use. Apart from the more extreme cases, stable recovery and reconstruction is generally achievable with moderate data amount which otherwise would be problematic under standard matched filtering. The performance levels are further improved significantly if longer pulses are put to use. Figures 7 show a comparative plot with the pulse length at ten samples where the target detection probability generally shifts for all type of block codes.

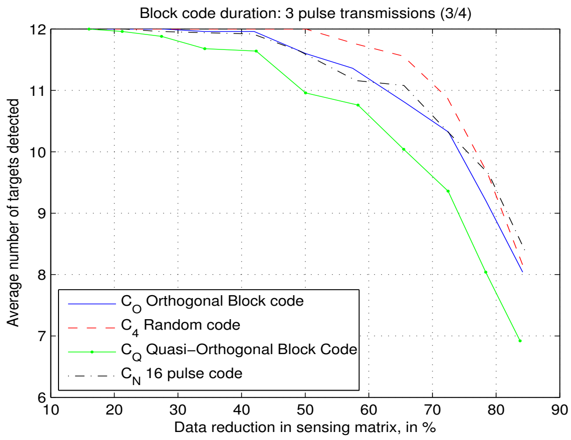

Fig. 7. Target detection with CS, 3 block code rows, SNR=8dB, $p_{len} = 10$

IV. Conclusion

This purpose of this paper has been to demonstrate a space-time block coding method for multistatic radar systems combined with a compressed sensing approach at the receiver. This use of block codes allows for flexibility in selecting arbitrary non-orthogonal waveforms while compressive sensing methodology permits a great deal of reduction in data acquisition through random data collection rather than continuous sampling and a reduction of the integration period of a block code. The paper has analyzed different block-codes and found that overall selecting arbitrary codes performs better than orthogonal block codes. It was shown that these approaches work well and provide estimates of the dominant targets even with significant data reduction. The utilization of compressed sensing with block codes follows the framework of similar methods where the hardware requirements can be diluted and instead replaced by advanced processing in software.

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